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On the Valuation of Property held for Life and in Reversion; and on the due Apportionment of it, when so held on the same Life, between the Tenant for Life and the Remainder-man. By CHARLES JELlicoe, one of the Vice Presidents of the Institute of Actuaries.

[Read before the Institute of Actuaries, 26th November, 1855, and ordered by the Council to be printed.]

IN a paper* published some time back in the *Journal* of the Institute, an endeavour was made to show the true principles upon which the value of securities depending on an isolated human life should be determined. Discarding the methods once so generally practised, and in accordance with which such securities were confounded with those depending on the average duration of many lives, it was urged that they should be regarded in a distinct point of view, and as affording the means of investments to be made not subject to any contingency whatever, but as securing to the holder a given rate of interest so long as he retains them, and the return of the capital invested when such interest shall cease to be paid or to accrue. In the case of an ordinary life interest, it was shown that these conditions are fully met by an estimate made in accordance with the simple formula $\frac{1}{d+p} - 1$; and in that of a reversion, by

* See article "On the contrivances required to render contingent reversionary interests marketable securities" (vol. ii., p. 159); also papers on the same subject by Robert Tucker, Esq. (vol. v., pp. 162 and 239.)

the equally convenient expression $1 - d(1 + A)$. Both of these point very clearly to obvious arrangements by which a purchaser can effectually secure the conditions above mentioned, and reduce properties, uncertain in their duration and of possibly remote realization, to a form in which they assume well defined and convenient characteristics.

The principles upon which these values should be estimated being thus laid down, I come now to consider the methods to be adopted when we are called upon to apportion a property held on the same life for life and in reversion, between the tenant in possession and the remainder-man (as, in legal phraseology, he is sometimes termed). A discussion of this subject is rendered the more necessary, from the fact that there exists a great diversity both of opinion and practice in regard to it, and that the solutions given in cases of the kind are so various as to excite no little surprise and distrust in the minds of those unacquainted with the subject, under whose notice such solutions are brought. The cause of these discrepancies I will now proceed to show, and in the course of the inquiry will endeavour to establish such a basis for the determination of these questions as may serve in some degree to obviate a recurrence of inconsistencies which operate injuriously in a public point of view, and are more or less prejudicial to the character of the profession.

The form in which the cases in question present themselves is generally a very simple one. A fund (usually consisting of Government stock) or an estate is to be divided between the tenant for life and the person entitled to the reversion, and the actuary's opinion is requested as to the value of the share of each in the fund. For the sake of illustration, we will suppose this last to be £1,000, Consols, and the age of the tenant for life to be 70. Now I believe the mode of solution most generally adopted is to assume the market rate of interest prevalent at the time, to look upon the fund as a perpetual annuity, and to deduct from the value of it that of the annuity during the life of A (the tenant in possession): that is to say, B's interest would be denoted by the formula

$$1. \left(\frac{1}{r} - A \right)$$

where r and A are taken, say, at 5 per cent. By this rule, the share of A will be £190, and that of B (the reversioner) £660, supposing the stock to realize £850.

Many actuaries, however, will regard the matter in quite an-

other point of view. They will look upon the reversioner's interest as consisting of a capital sum, receivable at decease of A; and, having provided for that at the market rate of the day, will consider the remainder of the fund as A's share. By this mode the value of A's share may be denoted by the expression

$$S \cdot (1 - V);$$

and as it is uncertain what the Consols will realize at A's death, it will be assumed that the average price will then prevail, and that the £1,000 stock will produce £850. Reasoning in this way, A's share will come out £337, and B's £513; and the latter will have the benefit or otherwise of any gain or loss attending the immediate sale of the stock, should such be to be made. If for V we substitute $1 - d(1 + A)$, taking d at 5 per cent.* and A at $3\frac{1}{2}$ per cent.—that is to say, at such a rate as an annuity on A's life could be purchased at in a respectable Society—we shall find A's share to be £319, and B's £531; and these are the sums they might severally realize, supposing the stock to be sold at the same rate as before.

But the sources of difference in these estimates are not yet exhausted. Some gentlemen of the profession will regard the question still in another light. They will argue that it is necessary only to place one of the parties in precisely the same position after the division as he occupied previously; and that, the portion of the fund required for this purpose being set aside, the remainder of it will of right belong to the other. As the original security is a Government one, they will ascertain the cost of an equivalent Government annuity for the life of A, and whatever the remainder of the fund will produce they will regard as the property of the reversioner; for the tenant for life, it will be urged, is neither a gainer nor a loser by the arrangement, and therefore the reversioner's interest remains in like manner unaffected by the change. Thus, supposing Consols to be at 85, A's share of the £1,000, stock, will be £258, since that will suffice to purchase an annuity of £30 during his life; and B will get £742, the remainder. In other words, the share of each, in ready money, will be £220 and £630, Consols being supposed to be at the price above quoted.

* When a reversion is valued as if it were the single premium for an assurance, and no provision made for interest during the life of the tenant, the rate of interest should evidently be higher than when the reverse is the case; in other words, a security upon which interest is immediately payable cannot be expected to yield to a purchaser so high a rate as one where it is altogether deferred: accordingly, d is here taken at 5 per cent. But in the foregoing example V is calculated at 6, for the reason here given.

Here then we have four ways of dividing the fund, and each of them attended with a different result.

By the first, the shares of A and B respectively will be	£190 and £660
By the second	£337 and £513
By the third	£319 and £531
By the fourth	£220 and £630

Now I shall endeavour to show that these methods are none of them strictly accurate, and to point out in what their faultiness consists. But first it will be convenient to lay down what I believe to be the true principles in accordance with which such questions should be solved, and from which, unless good reason can be shown to the contrary, it is inexpedient at any time to depart. It must be observed, then, that of the cases in question presenting themselves, some arise under circumstances different from those out of which others originate. Thus the life tenant and reversioner may be both desirous of realizing, and may be willing to unite their interests in order to effect that object with greater advantage; or one of the parties may wish to obtain possession of the whole fund, making compensation to the other for the abandonment of his interest in it, and for the trouble and inconvenience to him attending the change. Now, as regards the first condition, it is obvious that the interest of the parties are only not the same as they would be were they each selling separately, from the circumstance of their being enabled to offer the whole fund to one purchaser, and thus greatly to increase its value; and as this increased value may be considered as arising from an augmentation in the value of each share, it seems only reasonable that it should be apportioned accordingly. Hence the right mode of procedure in this case would seem to be, to estimate the interests of the life tenant and of the reversioner as they would be were they to be sold separately, and to divide between them the proceeds of the fund in the proportions indicated by the estimates thus made. But it has been shown that the market values of the interests in question are properly found by the formulæ

$\frac{1}{d+p} - 1$ and $1 - d(1 + A)$ where the current rates for the use of money are taken, and where the two interests are charged, as they should be, with the expense of an assurance in the one case and of an annuity in the other.

As the sum of these, then, is to the proceeds of the fund, so will the amount of either be to its share of it; and if we assume, as before, that the stock will realize £850, and adopt a practical rate of annuity and of premium, we shall obtain for the separate values

£152 and £530: whence the share of each in the total fund will be £190 and £660 respectively—a result which happens to be identical with that of the method first mentioned, but which, it will be seen, is arrived at in a very different way.

As the next case for our consideration, we will take the one in which the reversioner is desirous of becoming possessed of the stock, and the question arises as to the sum to be given to the life tenant for his interest in it. Here it is apparent that the latter should be placed as nearly as possible in the same situation after parting with his share as he was previously, and whatever it may cost to effect this the reversioner may fairly be called upon to pay. This case, then, is appropriately met by the last of the four methods I have described, since the life tenant will by the means therein used, be allotted as much as will enable him to retain as nearly as possible the same kind of security he had at first, and enjoy precisely the same income. It will be seen that the reversioner will have to pay £220 for the release given by the other of his interest, and thus become possessed of the entire fund.

The next and last condition which we have to investigate is that in which the owner of the life interest seeks to gain possession of the whole estate, and the consideration to be awarded to the reversioner has to be determined. Here, of course, the observations applied in the foregoing case to the life tenant adapt themselves to the reversioner; and, by parity of reasoning, his position must at least be preserved without prejudice. To effect this, he should have such a sum as will enable him without difficulty to purchase a similar reversion to that he is asked to surrender; and as he can only certainly get this at a moment's notice from an Assurance Company, it would appear that the tenant for life should give him sufficient to assure his reversion at A's death, the option remaining with him to carry out such a transaction or not as he might think fit; but the single premium to assure £850 at death, of a person aged 70, would be about £670, and this is therefore the sum which A should give to B for the surrender of his interest in the estate.

If the truth of these principles be admitted, we may now proceed to examine the methods described at the commencement of this paper, and to see in what respects they are applicable or otherwise to the cases in question. As regards the first, then, which I have indicated by the expression $I. \left(\frac{1}{r} - A \right)$, it appears to me to be altogether erroneous, from its dealing with B's interest as though it were a perpetual annuity. At A's death, B succeeds to the corpus

of the estate, and can dispose of it as he pleases: the assigning to him the perpetuity only is a restriction not justified by the fact. The property he succeeds to is liable to variation in its value, although the income from it be fixed; and this variation must be taken into account. Moreover, A's interest is incorrectly estimated. It is obvious that, in the market, the cost of insurance would have to be deducted, whereas that consideration is altogether omitted. This method, then, is applicable to neither of the conditions which have been discussed. The second one, $S \cdot (1 - V)$, is I think faulty, from its regarding A's interest as a sum receivable at A's death, without making any provision for payment of interest in the interval—an omission which, as I have already stated, appears to me to be unjustifiable. If $1 - d(1 + A)$ be substituted for V , the shares will be nearly what they should come out when the reversioner is asked to surrender his interest, and is to receive adequate compensation for it, but not sufficiently so to justify the use of the formula for that purpose; and it is, as will be seen, unsuited for the others I have mentioned. The last mode, then, is the only one which would appear to be in any case properly adapted to the object in view. It seems applicable when the making a fitting compensation to the life tenant for the surrender of his share of the fund is the case in question. Under any other circumstances the use of it would lead to erroneous results.

From what has been said, it will be apparent that, in questions of this kind, a great deal of judgment and discretion must always be required from the actuary, and that it is not possible to lay down any general rule which shall altogether obviate such requirement. Little else, after all, can be done, than to bring under notice the various arguments which apply in such cases, leaving it to those who are concerned to make the selection proper to the one they may have in hand. That they may be enabled to do this, it is obvious that the fullest information should be given, not only as to the security to be dealt with, but as to the position and objects of the parties concerned; and, thus provided, the actuary must execute his task as best he may, bearing in mind the ancient precept, *Est modus in rebus; sunt certi denique fines, quos ultra citraque nequit consistere rectum.*
